26.

$f(t) = \frac{2}{1-t^2}$ $\quad g(x) = \int_0^x f(t)\,dt$

geo $\frac{a}{1-r}$

a) $\frac{2}{1-t^2} = 2 + 2t^2 + 2t^4 + 2t^6 \ldots 2t^{2n}$

b) $g(x) = 2x + \frac{2x^3}{3} + \frac{2x^5}{5} + \frac{2x^7}{7} \ldots \frac{2x^{2n+1}}{2n+1}$

Feb 4-9:15 AM

43. $f(x) = \frac{\sin x}{x}$

Feb 4-9:22 AM

9.3 Taylor series with remainder

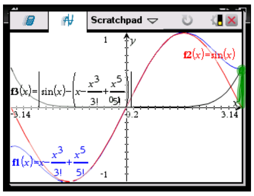
What is the fifth order Maclaurin series for f(x)=sin(x)? What is the maximum error when approximating sin(x) on $[-\pi,\pi]$

Solve graphically and numerically

series: $\underset{f_1(x)}{\sin x} \approx \underset{f_2(x)}{x - \frac{x^3}{3!} + \frac{x^5}{5!}}$

error = ϵ = $|f_1(x) - f_2(x)|$

$f_3(x) =$



.524

max error = .524

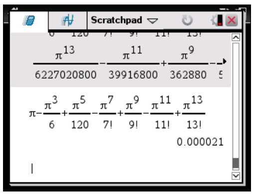
$f_3(\pi)$

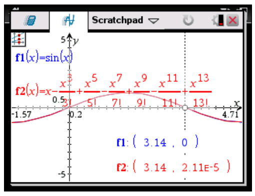
Jan 31-5:30 PM

How many terms are needed in the Maclaurin series for sin(x) in order to approximate sin(x) within .0001 on the interval $[-\pi,\pi]$

error

need the series - trial & error



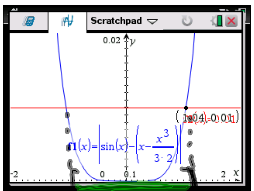


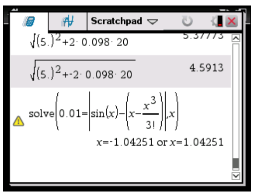
7 terms

Jan 31-6:00 PM

On what interval does the third order Maclaurin series approximate sin(x) within .01?

error





-1.0425 $\quad$ 1.0425

Jan 31-6:02 PM

Taylor's Remainder Estimation Theorem

$f(x) \approx f(a) + f'(a)(x-a) + \ldots + \frac{f^n(a)(x-a)^n}{n!}$

$\boxed{\epsilon \le \left| M \cdot \frac{(x-a)^{n+1}}{(n+1)!} \right|}$

M is max of $f^{n+1}(x)$

Jan 31-6:03 PM

The approximation $\ln(1+x) \approx x - \frac{x^2}{2}$ is used when x is small.

Use the Remainder Estimation Theorem to get a bound for the maximum error when $|x| \leq .01$ Support the answer graphically.

$-.01 \leq x \leq .01$

$\epsilon \leq \left| M \cdot \frac{x^3}{3!} \right|$ M is max of f'''

$f = \ln(1+x)$

$f' = \frac{1}{1+x} = (1+x)^{-1}$

$f'' = -1(1+x)^{-2}$

$f''' = 2(1+x)^{-3} = \frac{2}{(1+x)^3}$ graph on [-.01, .01

$M = \frac{2}{(1+-.01)^3} = 2.0612$

$x = .01$

$\epsilon < \left| 2.0612 \frac{(.01)^3}{3!} \right| = 3.435E-$

.00000343

Jan 31-6:07 PM